DOI: 10.2478/auom-2019-0014
An. Şt. Univ. Ovidius Constanţa **Vol. 27(1),2019, 269–288**

Pre-semihyperadditive Categories

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Abstract

In this paper we extend the notion of classical (pre-)semiadditive category to (pre-)semihyperadditive category. Algebraic hyperstructures are algebraic systems whose objects possessing the hyperoperations or multi-valued operation. We introduce categories in which for objects A and B, the class of all morphisms from A to B denoted by $Mor(A, B)$, admits an algebraic hyperstructures, such as semihypergroup or hypergroup. In this regards we introduce the various types of pre-semihyperadditive categories. Also, we construct some (pre-)semihyperadditive categories by introducing a class of hypermodules named general Krasner hypermodules. Finally, we investigate some properties of these categories.

1 Introduction

In 1934, the concept of *hypergroupoid* was introduced by Marty [23] based on a "hyperoperation" (or "hypercompositin") on a nonempty set. In fact, a hyperoperation on a nonempty set H is a mapping say $\circ$ on $H \times H$, which assigns to each pair (a, b) in $H \times H$ a nonempty subset $a \circ b$ of H. By this way we obtain a new algebraic system $(H, \circ)$ which is called a hypergroupoid. This definition has resulted in a new branch of mathematics named Hyperstructures Theory. This theory has been developed in view points of theory and applications by many researchers in this area (for more see [7, 10, 11, 13, 14, 16, 24, 25, 33]). One of the important area of algebraic hyperstructures is the interaction between hyperstructure theory and category theory [5, 8, 9, 15, 26–28].

Key Words: Pre-semihyperadditive Category, Pre-hyperadditive Category, Pre-canonical Hyperadditive Category, Hypercategory.
2010 Mathematics Subject Classification: Primary 20N20; Secondary 20N25.
Received: 25.07.2017
Accepted: 27.01.2018

It is well known a category $\mathcal{C}$ consists of a class of objects and a class of morphisms with some data ([2]). In the classical study of category theory for two objects A and B of category $\mathcal{C}$, the class of morphisms from A into B is denoted by $Mor_{\mathcal{C}}(A, B)$. Also, for some categories such as the category of abelian groups, $(Mor_{\mathcal{C}}(A, B), +)$ is an abelian group. But when we deal with the categories of hyperstructures, such as the category of hypergroups, with hypergroups as objects and usual homomorphisms as morphisms, $f + g$ does not belong to $Mor_{\mathcal{C}}(A, B)$, since $f(a) + g(a)$ is a nonempty subset of H. This leads us to investigate a suitable hyperoperations on $Mor_{\mathcal{C}}(A, B)$, say $\star$, such that the system $(Mor_{\mathcal{C}}(A, B), \star)$ admits a hyperstructure such as a semihypergroup or hypergroup.

This paper is divided as follows. In Section 2, we give some basic definitions and results of hyperstructure theory, which we need to develop our paper. In Section 3, first we investigate some hyperstructures on $Mor_{\mathcal{C}}(A, B)$ to generalize the notion of (pre-)semiadditive category to (pre-)semihyperadditive category (or hypercategory). Then by introducing some examples, we illustrate these categories. For example, we introduce some categories of hypermodules named general Krasner hypermodules denoted by ${}_R\mathcal{G}.\mathbf{mod}$ and ${}_R\mathbf{G}.\mathbf{mod}$ based on different kinds of morphisms and study some basic properties of these categories.

2 Preliminary

Let us recall some basic notions and definitions. H be a nonempty set and $P^*(H)$ be the set of all nonempty subsets of H. A hyperoperation $\cdot$ on H is a map

$$\cdot : H \times H \longrightarrow P^*(H), \qquad (a, b) \mapsto a \cdot b \subseteq H$$

Then $(H, \cdot)$ is called a hypergroupoid. The hyperoperation $\cdot$ is extended to subsets of H in a natural way, so that $A \cdot B$ or AB is given by

$$AB = \bigcup_{(a,b)\in A\times B} a \cdot b. \tag{2 .1}$$

A hypergroupoid $(H, \cdot)$ is said a semihypergroup if $\cdot$ is associative. A hypergroupoid $(H, \cdot)$ is said a hypergroup if it is a semihypergroup satisfying (*reproductivity* property) $Hx = xH = H$, for every $x \in H$. A semihypergroup or hypergroup H is called commutative if $xy = yx$ for every $x, y \in H$. An element y of semihypergroup $(H, +)$ is called an identity if for all $x \in H$, $y \in x + y \cap y + x$.

Definition 2.1. *Let x be an element of semihypergroup $(H,+)$ (resp., $(H,\cdot)$) such that $x+y=y$ (resp., $x\cdot y=y$). Then x is called a left scalar identity (resp., unit). Similarly, a right scalar identity (resp., unit) is defined by the affection on the right.*

An element x of semihypergroup $(H,+)$ (resp., $(H,\cdot)$) is called a scalar identity (resp., unit) if it is a left and right scalar identity (resp., unit). We denote the scalar identity (resp., unit) of H by 0_H (resp., 1_H). Every scalar identity or scalar unit in a semihypergroup H is unique.

Definition 2.2. *A nonempty set H together with the hyperoperation $+$ is called a canonical hypergroup if the following axioms hold:*

1. *$(H,+)$ is a commutative semihypergroup,*
2. *there is a scalar identity 0_H,*
3. *for every $x\in H$, there is a unique element denoted by $-x$ such that $0_H\in x+(-x)$ which for simplicity we write $0_H\in x-x$,*
4. *$x\in y+z$ implies $y\in x-z$ (and thus $z\in -y+x$).*

A nonempty subset K of a canonical hypergroup H is called a canonical subhypergroup denoted by $K\leq H$ if K is a canonical hypergroup itself.

Definition 2.3. [20] *A nonempty set R together with the hyperoperation $+$ and the operation $\cdot$ is called a Krasner hyperring if the following axioms hold:*

1. *$(R,+)$ is a canonical hypergroup (with scalar identity 0_R),*
2. *$(R,\cdot)$ is a semigroup including 0_R as a bilaterally absorbing element, i.e., $0_R\cdot a=a\cdot 0_R=0_R$ for all $a\in A$,*
3. *$(y+z)\cdot x=(y\cdot x)+(z\cdot x)$ and $x\cdot(y+z)=x\cdot y+x\cdot z$ for all $x,y,z\in R$.*

We say a Krasner hyperring $(R,+,\cdot)$ has 1_R if $(R,\cdot)$ has the (scalar) unit 1_R that is $1_R\cdot r=r\cdot 1_R=r$ for all $r\in R$.

Definition 2.4. [29] *Let R be a Krasner hyperring. A nonempty set A is called a Krasner hypermodule over R, for short a left Krasner R-hypermodule, if $(A,+)$ is a canonical hypergroup together with the map $*\colon R\times A\longrightarrow A$ satisfying the following axioms for all $r_1,r_2\in R$ and $a_1,a_2\in A$:*

1. $r_1*(a_1+a_2):=\bigcup\limits_{b\in a_1+a_2} r_1*b=r_1*a_1+r_1*a_2,$
2. $(r_1+r_2)*a_1:=\bigcup\limits_{r\in r_1+r_2} r*a_1=r_1*a_1+r_2*a_1,$

3. $(r_1 \cdot r_2) * a_1 = r_1 * (r_2 * a_1)$,

4. $0_R * a_1 = 0_A$.

A left Krasner R-hypermodule A is called *unitary* if R has 1_R and $1_R * a = a$ for all $a \in A$.

Example 2.5.

(i) Every left module over a ring R is a left Krasner R-hypermodule.

(ii) Every ring R is a left Krasner R-hypermodule.

(iii) Every Krasner hyperring R is a left Krasner R-hypermodule. As a special case, consider the set of nonnegative real numbers denoted by $\mathbb{R}^{+0}$. It is easy to verify that $\mathbb{R}^{+0}$ under the hyperoperation $+$ defined by $x + y = max\{x, y\}$ if $x \neq y$, and $[0, x]$ if $x = y$, is a canonical hypergroup. Defining $*\colon \mathbb{R}^{+0} \times \mathbb{R}^{+0} \longrightarrow \mathbb{R}^{+0}$ as the usual multiplication, we can check that $\mathbb{R}^{+0}$ satisfies all axioms mentioned in Definition 2.4. Consequently, $\mathbb{R}^{+0}$ is a unitary left Krasner $\mathbb{R}^{+0}$-hypermodule (see [32]).

(iv) Let $(R, +, \cdot)$ be a division ring and M be an R-module. Consider G as a normal subgroup of multiplicative semigroup $R \setminus \{0\}$. Define the equivalence relation ρ as follows:

$$x\rho y \Longleftrightarrow \exists t \in G : \quad x = ty.$$

Let $\bar{M} = M/\rho$ and $\bar{R} = R/G$. Then $\bar{M}$ together with $+'$ is a canonical hypergroup as follows (see [21]):

$$\forall \bar{x}, \bar{y} \in \bar{M} : \quad \bar{x} +' \bar{y} := \{\bar{z} \in \bar{M} \mid \bar{z} \subseteq \bar{x} + \bar{y}\}.$$

Also, $\bar{M}$ is a left Krasner $\bar{R}$-hypermodule by the external multiplication:

$$\forall \bar{r} \in \bar{R}, \forall \bar{x} \in \bar{M} : \quad \bar{r} * \bar{x} := \overline{rx}.$$

(v) Let $(R, +, \cdot)$ be a ring and N be a normal subgroup of semigroup $(R \setminus \{0\}, \cdot)$. Let $\bar{R} = R/N$ be the set of classes of the form $\bar{x} = x \cdot N$. If for all $\bar{x}, \bar{y} \in \bar{R}$, we define $\bar{x} +' \bar{y} = \{\bar{z} | \quad z \in \bar{x} + \bar{y}\}$, and $\bar{x} * \bar{y} = \overline{x \cdot y}$ as the external multiplication, then $\bar{R}$ is a left Krasner $\bar{R}$-hypermodule (for more details about hyperstructures, see [1, 6, 7, 16–18, 21, 29, 33]).

3 Category theory and hyperstructures theory

3.1 Categories motivated by hyperstructures

In category theory, a *pre-additive* category is a category $\mathcal{C}$ in which for all $A, B \in Ob(\mathcal{C})$, $(Mor_{\mathcal{C}}(A, B), +)$ is an abelian group and the composition $\circ$ is distributive with respect to the operation $+$ on the left and right, i.e.,

$$f \circ (g + h) = f \circ g + f \circ h$$

$$(g + h) \circ f = g \circ f + h \circ f.$$

Motivated by hyperstructures theory, we can define a hyperoperation on $Mor_{\mathcal{C}}(A, B)$.

In this section, we introduce some new categories in which $(Mor_{\mathcal{C}}(A, B), +)$ is a hyperstructure. First we start with the following concept.

Definition 3.1. *A category $\mathcal{C}$ is called pre-semihyperadditive, for short $\mathcal{PSHA}$, if for all $A, B \in Ob(\mathcal{C})$,*

1. *$(Mor_{\mathcal{C}}(A, B), \oplus)$ is a commutative semihypergroup having a scalar identity with:*

$$\oplus : Mor_{\mathcal{C}}(A, B) \times Mor_{\mathcal{C}}(A, B) \longrightarrow P^*(Mor_{\mathcal{C}}(A, B))$$

$$(f, g) \mapsto f \oplus g \subseteq Mor_{\mathcal{C}}(A, B);$$

2. *the composition $\circ$ is distributive with respect to the hyperaddition $\oplus$ on the left and right , i.e.,*

$$f \circ (g \oplus h) \subseteq f \circ g \oplus f \circ h,$$

$$(g \oplus h) \circ f \subseteq g \circ f \oplus h \circ f,$$

with definable domain and codomain for morphisms f, g, h;

3. *the zero morphism $0_{A,B} \in Mor_{\mathcal{C}}(A, B)$ satisfies $f \oplus 0_{A,B} = f = 0_{A,B} \oplus f$ for all $f \in Mor_{\mathcal{C}}(A, B)$.*

In second property of Definition 3.1, if the equality holds, then $\mathcal{C}$ is called *strong* pre-semihyperadditive, for short $s - \mathcal{PSHA}$.

Also, if

$$f \circ (g \oplus h) \cap f \circ g \oplus f \circ h \neq \emptyset,$$

$$(g \oplus h) \circ f \cap g \circ f \oplus h \circ f \neq \emptyset,$$

then $\mathcal{C}$ is called *weak* pre-semihyperadditive, for short $w - \mathcal{PSHA}$.

To avoid any confusion, sometimes we use $i - \mathcal{PHA}$ instead of $\mathcal{PHA}$.

Definition 3.2. *A $\mathcal{PSHA}$ (resp., $s - \mathcal{PSHA}$, $w - \mathcal{PSHA}$) category $\mathcal{C}$ in which $(Mor_{\mathcal{C}}(A, B), \oplus)$ is a*

(i) hypergroup is called a (resp., strong, weak) pre-hyperadditive category, for short $\mathcal{PHA}$ (resp., $s - \mathcal{PHA}$, $w - \mathcal{PHA}$) category.

(ii) canonical hypergroup is called a (resp., strong, weak) canonical pre-hyperadditive category, for short $\mathcal{CPHA}$ *(resp.,* $s-\mathcal{CPHA}$*,* $w-\mathcal{CPHA}$*) category.*

Remark 3.3. *In some categories, only the first containment of the second property of Definition 3.1 is considered and holds (see Theorem 3.22). We denote such categories by* $i_1-\mathcal{PSHA}$ *(resp.,* $s_1-\mathcal{PSHA}$*,* $w_1-\mathcal{PSHA}$*) depending on the first containment (resp., equality, nonempty intersection). Also,* $i_1-\mathcal{CPHA}$ *(resp.,* $s_1-\mathcal{CPHA}$*,* $w_1-\mathcal{CPHA}$*) categories are defined in an obvious way.*

We just considered some generalizations of (classical) category. But there is a different kind of such generalization and that is when $\circ$ is a hyperoperation. So we can generalize the above definitions and apply the word "*hypercategory*" instead of word "category" to show that we mean $\circ$ is a hyperoperation. Thus similarly one can define $\mathcal{PSHA}$, $\mathcal{PHA}$, and $\mathcal{CPHA}$ *hypercategory.* For example, one $s-\mathcal{CPHA}$ hypercategory is defined as follows:

Definition 3.4. *A* $s-\mathcal{CPHA}$ *hypercategory denoted by* $\mathfrak{C}$ *consists of*

1. *a class of objects:* $A, B, C, \ldots$
2. *a class of morphisms or arrows:* $f, g, h, \ldots$ *with the following data:*
   - *given morphisms* $f : A \longrightarrow B$ *and* $g : B \longrightarrow C$*, that is, with:* $cod(f) = dom(g)$ *there is given a "subset"* $g \circ f$ *of morphisms from* A *to* C *called the composition of morphisms* f *and* g*;*
   - $(Mor_{\mathfrak{C}}(A, B), \circ)$ *is a semihypergroup:* $h \circ (g \circ f) = (h \circ g) \circ f$ *for all* $f : A \longrightarrow B$*,* $g : B \longrightarrow C$ *and* $h : A \longrightarrow C$*;*
   - $(Mor_{\mathfrak{C}}(A, B), \circ)$ *has a scalar unit: for every two objects* A *and* B *and every morphism* $f \in Mor_{\mathfrak{C}}(A, B)$*, there are morphisms* id_A *and* id_B *such that* $f \circ id_A = f = id_B \circ f$ *for all* $f : A \longrightarrow B$*;*
3. $(Mor_{\mathfrak{C}}(A, B), \oplus)$ *is a canonical hypergroup having a scalar identity;*
4. *the composition* $\circ$ *is distributive with respect to the hyperaddition* $\oplus$ *on the left and right , i.e.,*

$$f \circ (g \oplus h) = f \circ g \oplus f \circ h,$$

$$(g \oplus h) \circ f = g \circ f \oplus h \circ f,$$

   with definable domain and codomain for morphisms f, g, h*;*

5. *the zero morphism* $0_{A,B} \in Mor_{\mathfrak{C}}(A, B)$ *satisfies* $f \oplus 0_{A,B} = f = 0_{A,B} \oplus f$ *for all* $f \in Mor_{\mathfrak{C}}(A, B)$*.*

For the complexity of hypercategories we leave the concept of hypercategory and only refer to an example of a hypercategory will be mentioned in Example 3.11.

Definition 3.5. *We say $\mathcal{C}$ has (finite) product (resp., coproduct), if every (finite) family of objects has (finite) product (resp., coproduct).*

Definition 3.6. *A pre-semihyperadditive category ($\mathcal{PSHA}$) is called semi-hyperadditive, for short $\mathcal{SHA}$ category if it has a zero object as well as finite product.*
Similarly, every $i_1 - \mathcal{PSHA}$ (resp., $s_1 - \mathcal{PSHA}$, $w_1 - \mathcal{PSHA}$, $\mathcal{CPHA}$, $i_1 - \mathcal{CPHA}$, $s_1 - \mathcal{CPHA}$, $w_1 - \mathcal{CPHA}$) category is called i_1-semihyperadditive (resp., s_1-semihyperadditive, w_1-semihyperadditive, canonical hyperadditive, i_1-canonical hyperadditive, s_1-canonical hyperadditive, w_1-canonical hyperadditive), for short $i_1 - \mathcal{SHA}$ (resp., $s_1 - \mathcal{SHA}$, $\mathcal{CHA}$, $i_1 - \mathcal{CHA}$, $s_1 - \mathcal{CHA}$, $w_1 - \mathcal{CHA}$) category if it has a zero object as well as finite product.

3.2 Examples in pre-semihyperadditive categories

Now we are chiefly planning to exhibit some examples in new categories mentioned in the previous part by introducing a new class of hypermodules. Before giving some examples, it is necessary to emphasize the condition 3 of Definition 3.1 by an example. The following example is a sample of a category that is not $\mathcal{PSHA}$.

Example 3.7. *It is easy to check that the category with only one object X together with the set of morphisms $\{id_X, 0, b\}$ equipped with the (hyper)operations defined by*

$+$	id_X	0	b
id_X	b	0	id_X
0	0	0	0
b	id_X	0	b

and

$\circ$	id_X	0	b
id_X	id_X	0	b
0	0	0	0
b	b	0	b

is a category that is not $\mathcal{PSHA}$ since the zero morphism 0 is not an identity element of $(Mor_{\mathcal{C}}(X,X), +)$. Indeed, $(Mor_{\mathcal{C}}(X,X), +)$ is a commutative monoid with identity b and the third condition of Definition 3.1 is not satisfied.

Example 3.8. *As the first available example, any Krasner hyperring* $(R, +, \cdot)$ *with* 1_R *can be seen as a* $\mathcal{PSHA}$ *category having a unique object, written* $\star$, *together with the elements of* R *as morphisms. Clearly,* $(Mor(\star,\star), +)$ *is a semihypergroup (canonical hypergroup). Also, by taking* $\cdot$ *as the composition of morphisms, clearly,* $\cdot$ *is left and right distributive with respect to* $+$. *The element* 1_R *acts as the unit morphism of this category. Clearly,* 0_R *is the scalar identity of* $(Mor(\star,\star), +)$ *and the zero morphism is* 0_R. *Moreover, this category is a* $s-\mathcal{PHA}$ *category as well as a* $s-\mathcal{CPHA}$ *category.*

To give another example, we introduce the following concept as a generalization of a Krasner hyperring.

Definition 3.9. *A nonempty set* R *together with two hyperoperations* $+$ *and* $\cdot$ *is called a general Krasner hyperring if the following axioms hold:*

1. $(R, +)$ *is a canonical hypergroup (with scalar identity* 0_R*),*
2. $(R, \cdot)$ *is a semihypergroup including* 0_R *as a bilaterally absorbing element, i.e.,* $0_R \cdot a = a \cdot 0_R = 0_R$ *for all* $a \in A$,
3. $(y+z)\cdot x \subseteq (y\cdot x)+(z\cdot x)$ *and* $x\cdot(y+z) \subseteq x\cdot y + x\cdot z$ *for all* $x, y, z \in R$.

We say a general Krasner hyperring $(R, +, \cdot)$ has the scalar unit 1_R if $1_R \cdot r = r \cdot 1_R = r$ for all $r \in R$.

In what follows, by a Krasner hyperring we construct a general Krasner hyperring in which the inclusions above are equality.

Example 3.10.

(i) Let $(R, +, \cdot)$ *be a Krasner ring with* 1_R. *Suppose* $R^{\mathbb{N}}$ *be the set of all sequences of the form* $a := (a_0, a_1, a_2, \ldots)$ *where* $a_i \in R$ *for all* $i \in \mathbb{N}\cup\{0\}$. *Define hyperoperations* $+'$ *and* $\cdot'$ *on* $R^{\mathbb{N}}$ *as follows:*
$a+'b = (a_0+b_0, a_1+b_1, \ldots)$ *and* $a\cdot' b$ *is all* $(c_0, c_1, \ldots)$ *where* $c_j \in \sum_{k+l=j} a_k\cdot b_l$ *for* $j \in \mathbb{N}\cup\{0\}$. *In what follows,* $(a+'b)_i$ *is the set of all* $c_i \in a_i + b_i$, *i.e.,* $(a+'b)_i = a_i + b_i$. *Also, by* $(a\cdot' b)_j$ *we mean the set of all* c_j *which* $c_j \in \sum_{k+l=j} a_k\cdot b_l$ *that simply we write* $(a\cdot' b)_j = \sum_{k+l=j} a_k \cdot b_l$.

Immediately, $(R^{\mathbb{N}}, +')$ *is a canonical hypergroup with scalar identity* $0 = (0_R, 0_R, \ldots)$.

Also, $(R^{\mathbb{N}}, \cdot')$ *is a semihypergroup in which* 0 *acts as an absorbing element. Suppose that* $a, b, c \in R^{\mathbb{N}}$, *and* $i \in \mathbb{N}\cup\{0\}$. *Then*

$$((a+'b)\cdot' c)_j = \sum_{k+l=j} (a+'b)_k \cdot c_l$$

$$= \sum_{k+l=j} (a_k + b_k)\cdot c_l = \sum_{k+l=j} (a_k\cdot c_l + b_k \cdot c_l)$$

$$= \sum_{k+l=j} a_k \cdot c_l + \sum_{k+l=j} b_k \cdot c_l = (a \cdot' c)_j +' (b \cdot' c)_j.$$

This shows that $(a +' b) \cdot' c = a \cdot' c +' b \cdot' c$. *By a similar argument one can prove* $c \cdot' (a +' b) = c \cdot' a +' c \cdot' b$ *and the associativity of* $\cdot'$ *on* $R^{\mathbb{N}}$.

$$(a \cdot' (b \cdot' c))_j = \sum_{k+l=j} a_k \cdot [(b \cdot' c)]_l$$

$$= \sum_{k+l=j} a_k \cdot [\sum_{r+s=l} b_r \cdot c_s] = \sum_{k+l=j} \sum_{r+s=l} a_k \cdot (b_r \cdot c_s).$$

By a simple calculation we have $(a \cdot' (b \cdot' c))_j = \sum_{k+r+s=j} a_k \cdot (b_r \cdot c_s)$. *Similarly, we obtain* $((a \cdot' b) \cdot' c)_j = \sum_{k+r+s=j} (a_k \cdot b_r) \cdot c_s$. *Since* $(R, \cdot)$ *is associative, we have* $(a \cdot' (b \cdot' c))_j = ((a \cdot' b) \cdot' c)_j$. *Consequently,* $a \cdot' (b \cdot' c) = (a \cdot' b) \cdot' c$.
So $(R^{\mathbb{N}}, +', \cdot')$ *is a general Krasner hyperring. Clearly,* $1 = (1_R, 0_R, 0_R, \ldots)$ *is the scalar unit of* $R^{\mathbb{N}}$. *This general Krasner hyperring is well known as* $R[[x]]$. *Similarly, the set all elements* $a := (a_0, a_1, a_2, \ldots) \in R^{\mathbb{N}}$ *where* $a_i = 0_R$ *for all but finitely* $i \in \mathbb{N} \cup \{0\}$ *denoted by* $R^{(\mathbb{N})}$ *forms a general Krasner hyperring well known as* $R[x]$.

(ii) Let $(R, +, \cdot)$ *be a Krasner hyperring with* 1_R *and (for* $n \in \mathbb{N}$*)* $M_n(R)$ *be the collection of all matrices of size* $n \times n$ *over* R. *Then, it is easy to check that* $M_n(R)$ *is a general Krasner hyperring with the scalar unit* $(a_{ij})_{n \times n}$ *where* $a_{ij} = 1_R$ *if* $i = j$, *otherwise* $a_{ij} = 0_R$.

Example 3.11. *Similar to Example 3.8, any general Krasner hyperring with* 1_R *leads to a "hypercategory". Moreover, this hypercategory is* $s - \mathcal{CPHA}$ *(see Definition 3.4).*

In the sequel, we are interested to give examples of categories introduced above by some hypermodules. Among the various types of hypermodules over a hyperring, we chiefly work with a kind of hypermodules (Definition 3.12) inspired by the definition of general Krasner hyperring. We call such hypermodules "*left general Krasner hypermodules*". Indeed, the following concept is the generalization of Definition 2.4.

Definition 3.12. *Let* R *be a general Krasner hyperring. A nonempty set* A *is called a general Krasner hypermodule over* R*, for short a left general Krasner* R*-hypermodule, if* $(A, +)$ *is a canonical hypergroup together with the map* $* : R \times A \longrightarrow P^*(A)$ *satisfying the following axioms for all* $r_1, r_2 \in R$ *and* $a_1, a_2 \in A$*:*

1. $r_1 * (a_1 + a_2) := \bigcup_{b \in a_1 + a_2} r_1 * b \subseteq r_1 * a_1 + r_1 * a_2,$

2. $(r_1 + r_2) * a_1 := \bigcup_{r \in r_1 + r_2} r * a_1 \subseteq r_1 * a_1 + r_2 * a_1$,

3. *if* $(r_1 \cdot r_2) * a_1 := \bigcup_{r \in r_1 \cdot r_2} r * a_1$ *and* $r_1 * (r_2 * a_1) := \bigcup_{a \in r_2 * a_1} r * a$, *then* $(r_1 \cdot r_2) * a_1 \subseteq r_1 * (r_2 * a_1)$,

4. $0_R * a_1 = 0_A$.

A general Krasner R-hypermodule A is called unitary if R has the scalar unit 1_R with $1_R * a = a$ for all $a \in A$. Every general Krasner hyperring R with 1_R is a unitary left general Krasner R-hypermodule.

As a motivation for Definition 3.12, note that an abelian group can be considered a unitary $\mathbb{Z}$-module. Inspired by this fact, one may tempt to generalize a similar statement for a canonical hypergroup to obtain a unitary left Krasner $\mathbb{Z}$-hypermodule (Definition 2.4). But this may not be true in general. For example, let $(A, +)$ be a canonical hypergroup and $|a - a| \geq 2$ for some $a \in A$. Then by the second axiom of Definition 2.4, we obtain the contradiction $a - a = (1 - 1) * a = 0 * a = 0_A$.

Remark 3.13. *In an obvious way, one can consider the external multiplication map* $*\colon A \times R \longrightarrow P^*(A)$ *to define the right general Krasner* R*-hypermodule. From now on,* R *denotes a general Krasner hyperring. Also, for convenience, by hyperring* R *and an* R*-hypermodule we mean a general Krasner hyperring and a left general Krasner* R*-hypermodule, respectively.*

In order to have a category whose objects are the class of all R-hypermodules, we need morphisms. For this, we start with the following concept.

Definition 3.14. *For two* R*-hypermodules* A *and* B*, let* f *be a function from* A *into* $P^*(B)$ *that satisfies the conditions*

1. $f(x + y) \subseteq f(x) + f(y)$,

2. $f(r * x) \subseteq r * f(x)$,

for all $r \in R$ *and all* $x, y \in A$*. In this case,* f *is said a multi-valued* R*-homomorphism, for short* $R - mv$*-homomorphism from* A *to* B*. Sometimes an* $R - mv$*-homomorphism is called an inclusion* $R - mv$*-homomorphism.*

Note that $+$ in Definition 3.14 is given by (2 .1). If f satisfies the conditions

1. $f(x + y) = f(x) + f(y)$,
2. $f(r * x) = r * f(x)$,

for all $r \in R$ and all $x, y \in A$, then it is called a *strong* $R - mv$*-homomorphism.* Also, if f satisfies the conditions

1. $f(x+y) \cap [f(x)+f(y)] \neq \emptyset$,
2. $f(r*x) \cap r*f(x) \neq \emptyset$,

for all $r \in R$ and all $x, y \in A$, then f is said to be a weak $R-mv$-homomorphism. The class of all $R-mv$-homomorphisms, strong $R-mv$-homomorphisms and weak $R-mv$-homomorphisms from A to B as morphisms from A to B is denoted by $Hom_R(A,B)$, $Hom_R^s(A,B)$ and $Hom_R^w(A,B)$, respectively.

Let $f \in Hom_R(A,B)$ and $g \in Hom_R(B,C)$. Define the composition $g \circ f$ as:

$$(g \circ f)(a) = \bigcup_{b \in f(a)} g(b), \quad \forall a \in A. \tag{3 .2}$$

Throughout the paper, ${}_R\mathcal{G}.\mathbf{mod}$, ${}_{R_s}\mathcal{G}.\mathbf{mod}$ and ${}_{R_w}\mathcal{G}.\mathbf{mod}$ denote the categories formed by the class of all R-hypermodules together with the class of all $R-mv$-homomorphisms, strong $R-mv$-homomorphisms and weak $R-mv$-homomorphisms, respectively, with the composition of morphisms as (3 .2).

One can consider a function f from A into B satisfying two conditions in Definition 3.14 as a morphism. We call such morphism an (inclusion) R-homomorphism. Similarly, we can define strong R-homomorphisms and weak R-homomorphisms from A to B. We use $hom_R(A,B)$, $\mathrm{hom}_R^s(A,B)$ and $hom_R^w(A,B)$ for the class of (inclusion) R-homomorphisms, strong R-homomorphisms and weak R-homomorphisms from A to B, respectively. Also, we denote the corresponding categories by ${}_R\mathbf{G}.\mathbf{mod}$, ${}_{R_s}\mathbf{G}.\mathbf{mod}$ and ${}_{R_w}\mathbf{G}.\mathbf{mod}$, respectively.
Any singleton set is identified with its element. Thus we may write $f(a)=b$ instead of $f(a)=\{b\}$. Therefore, any single-valued $f \in Hom_R(A,B)$ is an element of $hom_R(A,B)$, and conversely, any element of $hom_R(A,B)$ is a single-valued element of $Hom_R(A,B)$.

Let $f, g \in Hom_R(A,B)$. Define the relation $\leq$ on $Hom_R(A,B)$ in which $f \leq g$ means $f(x) \subseteq g(x)$ for all $x \in A$. Clearly $(Hom_R(A,B), \leq)$ is a poset.

Let $f, g, h \in Hom_R(A,B)$. Define the formal operation $+$ on $Hom_R(A,B)$ as

$$+ : Hom_R(A,B) \times Hom_R(A,B) \longrightarrow Hom_R(A,B)$$

$$(f,g) \longmapsto f+g$$

with $(f+g)(x) := f(x)+g(x)$ for all $x \in A$.

Note that the hyperoperation $+$ in $f(x)+g(x)$ is indeed the hyperoperation of canonical hypergroup $(B,+)$ that for convenience, we identify it with the formal notation $+$ in $f+g$.

Remark 3.15. *Let $f, g \in hom_R(A,B)$. Then $f+g$ sends an element $x \in A$ to an element of $P^*(B)$. Thus we can think of $hom_R(A,B)$ as a motivation for $Hom_R(A,B)$. Indeed, R-homomorphisms imply $R-mv$-homomorphisms.*

Clearly, $h \leq f+g$ if and only if $h(x) \subseteq f(x)+g(x)$ for all $x \in A$. Now define the hyperoperation $\uplus$ on $Hom_R(A,B)$ as follows:

$$f \uplus g = \{h \in Hom_R(A,B) \mid \quad h \leq f+g\},$$

or equivalently,

$$f \uplus g = \{h \in Hom_R(A,B) \mid \quad h(x) \subseteq f(x)+g(x) \quad \forall x \in A\}.$$

Note that the hyperoperation $\uplus$ on $hom_R(A,B)$ is reduced to the following:

$$f \uplus g = \{h \in hom_R(A,B) \mid \quad h(x) \in f(x)+g(x) \quad \forall x \in A\}.$$

Notation 3.16. *We denote the category of all R-hypermodules whose morphisms are all single-valued R-homomorphisms by the notation ${}_R\mathbf{G}.\mathbf{mod}$.*

Proposition 3.17.

(i) The categories ${}_R\mathcal{G}.\mathbf{mod}$ and ${}_R\mathbf{G}.\mathbf{mod}$ have the zero object.

(ii) For every $R-mv$-homomorphism $f \in Hom_R(A,B)$, $f(0_R) = 0_B$.

Proof.

(i) Define $\mathbf{0} := \{0\}$ with $0+0 = \{0\}$ and $r * 0 = \{0\}$ for every $r \in R$.

(i) From the fourth condition of Definition 3.12 together with $f(r*x) \subseteq r*f(x)$, we have $f(0_A) = 0_B$, for every morphism $f \in Hom_R(A,B)$. Indeed if $a \in A$, then $0_R * a = 0_A$ implies $f(0_A) = f(0_R * a) \subseteq 0_R * f(a)$. On the other hand, $0_R * f(a) \subseteq 0_R * B$ and $0_R * B = \{0_B\}$ imply $0_R * f(a) = \{0_B\}$. So $f(0_A) = 0_B$. $\square$

Remark 3.18. *Every morphism from A to B in ${}_R\mathcal{G}.\mathbf{mod}$ is an $R-mv$-homomorphism $f \in Hom_R(A,B)$ or a multi-valued function from A into B as mentioned in Definition 3.14. So for convenience, when speaking of morphisms (arrows) and dealing with diagrams, by a morphism $f\colon A \longrightarrow B$ in ${}_R\mathcal{G}.\mathbf{mod}$, we mean a function from A into $P^*(B)$ as Definition 3.14.*

Proposition 3.19.

1. *In the category ${}_R\mathcal{G}.\mathbf{mod}$, $Hom_R(A,B)$ is nonempty and $(Hom_R(A,B), \uplus)$ has a scalar identity that is a zero morphism.*

2. *In the category ${}_R\mathbf{G}.\mathbf{mod}$, $hom_R(A,B)$ is nonempty and $(hom_R(A,B), \uplus)$ has a scalar identity that is a zero morphism.*

Proof. Note that $0_A \in A \neq \emptyset \neq B \ni 0_B$. Consider the morphism $0\colon A \longrightarrow B$ given by $0(a) = 0_B$ for every $a \in A$. Clearly, we have $g \circ 0(a) = 0(a)$ for every $g \in Hom_R(B, C)$. So $g \circ 0 = 0$. Also, $0 \circ h = 0$ for every $h \in Hom_R(C, A)$. Hence 0 is a zero morphism denoted by $0_{A,B}$.
On the other hand, for every $f \in Hom_R(A, B)$ (or $f \in hom_R(A, B)$), we have

$$f(a) = 0_{A,B}(a) + f(a) = f(a) + 0_{A,B}(a).$$

Thus $f = 0_{A,B} \uplus f = f \uplus 0_{A,B}$. So the zero morphism $0_{A,B}$ acts as a scalar identity in $(Hom_R(A, B), \uplus)$ (or $(hom_R(A, B), \uplus)$). □

Now for an element $f \in Hom_R(A, B)$, we encounter a question related to defining $-f$ as follows.

Let $x \in A$ and $-x \in A$ be its inverse and $f \in Hom_R(A, B)$. Are two definitions $(-f)(x) := -f(x)$ and $(-f)(x) := f(-x)$ the same?

Clearly for every $f \in Hom_R(A, B)$, we have

$$-f(x) = \{-y \in B \mid \quad y \in f(x)\},$$

$$f(-x) = \{y \in B \mid \quad y \in f(-x)\}.$$

Now according to Proposition 3.17 and since f is an $R - mv$-homomorphism, $0_A \subseteq x + (-x)$ implies

$$0_B = f(0_A) \subseteq f(x + (-x)) \subseteq f(x) + f(-x).$$

So there are $b_1 \in f(x)$ and $b_2 \in f(-x)$ such that $0_B \in b_1 + b_2$. From the reversibility axiom $b_2 \in -b_1 + 0_B$ and thus $b_2 = -b_1$. This means that $-f(x) \cap f(-x) \neq \emptyset$. Although we do not answer the question, certainly $-f(x) \cap f(-x) \neq \emptyset$ implies the following statement.

Proposition 3.20. *Let $f \in Hom_R(A, B)$ be such that $|f(x)| = 1$ for all $x \in A$. Then $(-f)(x) := f(-x)$ is equivalent to $(-f)(x) := -f(x)$. In particular, for every $f \in hom_R(A, B)$ the result is true.*

Proposition 3.21. *Let A and B be two R-hypermodules. Then*

(i) $(hom_R(A, B), \uplus)$ *is a canonical hypergroup.*

(ii) $(Hom_R(A, B), \uplus)$ *is a semihypergroup with a scalar identity.*

Proof.

(i) The associativity of $\uplus$ is obtained from the associativity of $+$ in A. More precisely, if $f, g, h \in hom_R(A, B)$, then

$$\begin{aligned}
(f \uplus g) \uplus h &= \bigcup_{u \in f \uplus g} \{k \in hom_R(A, B) \mid \quad k \leq u + h\} \\
&= \bigcup_{u \in f \uplus g} \{k \in hom_R(A, B) \mid \quad k(x) \subseteq u(x) + h(x)\} \\
&= \{k \in hom_R(A, B) \mid \quad k(x) \subseteq (f(x) + g(x)) + h(x)\} \\
&= \{k \in hom_R(A, B) \mid \quad k(x) \subseteq f(x) + (g(x) + h(x))\} \\
&= \bigcup_{v \in g \uplus h} \{k \in hom_R(A, B) \mid \quad k(x) \subseteq f(x) + v(x)\} \\
&= \bigcup_{v \in g \uplus h} \{k \in hom_R(A, B) \mid \quad k \leq f + v\} = f \uplus (g \uplus h).
\end{aligned}$$

Commutativity of $hom_R(A, B)$ similarly follows from $f(x)+g(x) = g(x)+f(x)$ for all $x \in A$. According to (existing the scalar identity 0 in) Proposition 3.19, it is enough to prove that there exists a unique inverse element $-f$ for every $f \in hom_R(A, B)$, and also $(hom_R(A, B), \uplus)$ is reversible.
Let $a \in A$ and $f \in hom_R(A, B)$. If $y = f(a) \in B$, then $-y = -f(a)$. Thus

$$0_B \in y + (-y) = f(a) + (-f(a)) \quad \text{and} \quad 0_B \in (-y) + y = (-f(a)) + f(a)$$

Equivalently, by Proposition 3.20

$$\begin{aligned}
& 0_{A,B}(a) \in f(a) + (-f)(a) \quad \text{and} \quad 0_{A,B}(a) \in (-f)(a) + f(a) \\
\Longleftrightarrow\ & 0_{A,B}(a) \in [f + (-f)](a) \quad \text{and} \quad 0_{A,B}(a) \in [(-f) + f](a) \\
\Longleftrightarrow\ & 0_{A,B} \leq f + (-f) \quad \text{and} \quad 0_{A,B} \leq (-f) + f \\
\Longleftrightarrow\ & 0_{A,B} \in f \uplus (-f) \quad \text{and} \quad 0_{A,B} \in (-f) \uplus f.
\end{aligned}$$

The uniqueness of $-f$ is followed by the uniqueness of inverse of y in B.
Let $a \in A$, $f, g, h \in hom_R(A, B)$ and suppose $f \in g \uplus h$. Then clearly, $f(a) \in g(a) + h(a)$ implies $h(a) \in g(a) - f(a) = g(a) + (-h)(a)$. Thus $h \in g \uplus (-f)$ and $(hom_R(A, B), \uplus)$ is reversible.

(ii) It is similarly proved. □

Thus we have the following result:

Theorem 3.22. *Let R be a hyperring. Then*

(i) The category ${}_R\mathbf{G.mod}$ is $i_1 - \mathcal{CPHA}$.

(ii) The category ${}_R\mathcal{G}.\mathbf{mod}$ *is* $i_1 - \mathcal{PSHA}$*.*

(iii) The category ${}_{R_s}\mathbf{G}.\mathbf{mod}$ *is* $i_1 - \mathcal{CPHA}$*.*

(iv) The category ${}_{R_s}\mathcal{G}.\mathbf{mod}$ *is* $i_1 - \mathcal{PSHA}$*.*

Proof. We only prove items (i) and (ii). The rest of proof is similar.
Let A and B be two R-hypermodules. By Proposition 3.21, $(hom_R(A,B), \uplus)$ (resp., $(Hom_R(A,B), \uplus)$) is a canonical hypergroup (resp., semihypergroup with scalar identity).
Now we prove $f \circ (g \uplus h) \subseteq f \circ g \uplus f \circ h$ for all definable R-homomorphisms (resp., $R - mv$-homomorphisms) f, g and h. Let $k \in f \circ (g \uplus h)$. Then there exists some $k' \in g \uplus h$ such that $k = f \circ k'$. Let x be an arbitrary element of domain of k'. Since $k'(x) \subseteq g(x) + h(x)$ and f is an R-homomorphism (resp., $R - mv$-homomorphism), we get

$$\begin{aligned} k(x) = f(k'(x)) &\subseteq f(g(x) + h(x)) \\ &\subseteq f(g(x)) + f(h(x)) \\ &= (f \circ g)(x) + (f \circ h)(x). \end{aligned}$$

Thus according to the definition of $\uplus$, $k \in f \circ g \uplus f \circ h$ and the result is clear. □

Remark 3.23. *Recall that a left (resp., right) pre-semiring ([19]) is a nonempty set S together with operations $+$ and $\cdot$ such that $(S, +)$ and $(S, \cdot)$ are semigroups having the property $a \cdot (b+c) = a \cdot b + a \cdot c$ (resp., $(b+c) \cdot a = b \cdot a + c \cdot a$) for all $a, b, c \in S$. According to the proof of Theorem 3.22, $(Hom_R(A,B), \uplus)$ has a limited and more general structure rather than the classical case. Indeed, the operation $\circ$ on semihypergroup $(Hom_R(A,B), \uplus)$ with property $f \circ (g \uplus h) \subseteq f \circ g \uplus f \circ h$ gives us a new hyperstructure that we call "left pre-semihyperring".*

Proposition 3.24. *All categories* ${}_R\mathcal{G}.\mathbf{mod}$, ${}_{R_s}\mathcal{G}.\mathbf{mod}$, ${}_R\mathbf{G}.\mathbf{mod}$ *and* ${}_{R_s}\mathbf{G}.\mathbf{mod}$ *have product.*

Proof. Let $\{A_k\}_{k \in I}$ be a family of R-hypermodules of category ${}_R\mathcal{G}.\mathbf{mod}$. Consider the cartesian product $\prod_{k \in I} A_k := \{(a_k)_{k \in I} \mid a_k \in A_k\}$. Clearly, $\prod_{k \in I} A_k$ is an R-hypermodule. We claim that $\prod_{k \in I} A_k$ with the (natural) projection R-homomorphisms $\pi_j \colon \prod_{k \in I} A_k \longrightarrow A_j$ is the product of $\{A_k\}_{k \in I}$.

$$\begin{tikzcd} & A \arrow[dl, "f_j"'] \arrow[d, dotted, "f"] \arrow[dr, "f_{j'}"] & \\ A_j & \prod_{k \in I} A_k \arrow[l, "\pi_j"'] \arrow[r, "\pi_{j'}"] & A_{j'} \end{tikzcd}$$

Indeed, if we define $f(a) := (f_k(a))_{k \in I}$ (in which $f_k(a) \subseteq A_k$), then clearly, $f \in Hom_R(A, \prod_{k \in I} A_k)$ and $\pi_j \circ f = f_j$.

Let $f' \in Hom_R(A, \prod_{k\in I} A_k)$ with $f'(a) = (A'_k)_{k\in I}$ for $a \in A$ and some $A'_k \subseteq A_k$ (as another morphism in ${}_R\mathcal{G}.\mathbf{mod}$) such that $\pi_j \circ f' = f_j$. Then

$$A'_j = \pi_j((A'_k)_{k\in I}) = \pi_j(f'(a)) = f_j(a)$$

for every $j \in I$. So

$$f'(a) = (A'_k)_{k\in I} = (f_k(a))_{k\in I} = f(a).$$

Hence $f' = f$ and f is unique. So ${}_R\mathcal{G}.\mathbf{mod}$ has product.
Similarly, ${}_{R_s}\mathcal{G}.\mathbf{mod}$, ${}_R\mathbf{G}.\mathbf{mod}$ and ${}_{R_s}\mathbf{G}.\mathbf{mod}$ have product. □

Thus, according to Definition 3.6, Theorem 3.22, and Proposition 3.24, we have the following result:

Theorem 3.25. *Let R be a hyperring. Then*

(i) Both ${}_R\mathbf{G}.\mathbf{mod}$ and ${}_{R_s}\mathbf{G}.\mathbf{mod}$ are $i_1 - \mathcal{CHA}$.

(ii) Both ${}_R\mathcal{G}.\mathbf{mod}$ and ${}_{R_s}\mathcal{G}.\mathbf{mod}$ are $i_1 - \mathcal{SHA}$.

Proposition 3.26. *Both categories ${}_{R_s}\mathcal{G}.\mathbf{mod}$ and ${}_{R_s}\mathbf{G}.\mathbf{mod}$ have coproduct.*

Proof. Let $\{A_k\}_{k\in I}$ be a family of R-hypermodules of category ${}_{R_s}\mathcal{G}.\mathbf{mod}$. Then we claim that the coproduct of $\{A_k\}_{k\in I}$ is the R-hypermodule

$$\coprod_{k\in I} A_k := \{(a_k)_{k\in I} \in \prod_{k\in I} A_k \mid a_k = 0_{A_k} \quad \text{for all but finite} \quad k\}$$

with the (natural) injection R-homomorphisms $i_j\colon A_j \longrightarrow \coprod_{k\in I} A_k$.

$$\begin{tikzcd}
A_j \arrow[r, "i_j"] \arrow[dr, "f_j"'] & \coprod_{k\in I} A_k \arrow[d, dotted, "f"] & A_{j'} \arrow[l, "i_{j'}"'] \arrow[dl, "f_{j'}"] \\
 & A &
\end{tikzcd}$$

Indeed, if we define $f((a_k)_{k\in I}) := \sum_{k\in I} f_k(a_k)$, then clearly, $f \in Hom^s_R(\coprod_{k\in I} A_k, A)$ and

$$f(i_j(a_j)) = f((0,0,\ldots,0,a_j,0,\ldots,0)) = f_j(a_j) + \sum_{j\neq k\in I} f_k(0) = f_j(a_j)$$

implies $f \circ i_j = f_j$.
Let $f' \in Hom^s_R(\coprod_{k\in I} A_k, A)$ (as another morphism in ${}_{R_s}\mathcal{G}.\mathbf{mod}$) such that $f' \circ i_j = f_j$. Then

$$f'((a_k)_{k\in I}) = f'(\sum_{k\in I} i_k(a_k)) = \sum_{k\in I} f'(i_k(a_k)) = \sum_{k\in I} f_k(a_k) = f((a_k)_{k\in I}).$$

Hence $f' = f$ and f is unique. So ${}_{R_s}\mathcal{G}.\mathbf{mod}$ has coproduct.
Similarly, ${}_{R_s}\mathbf{G}.\mathbf{mod}$ has coproduct. $\square$

In the end, for completeness, we state the following straightforward results in which "$\preceq$" means " *is subcategory of*".

Theorem 3.27. *Let R be a hyperring. Then*

$$\begin{aligned}
{}_{R}\mathbf{G}.\mathbf{mod} &\preceq {}_{R}\mathcal{G}.\mathbf{mod},\\
{}_{R_s}\mathbf{G}.\mathbf{mod} &\preceq {}_{R_s}\mathcal{G}.\mathbf{mod},\\
{}_{R_w}\mathbf{G}.\mathbf{mod} &\preceq {}_{R_w}\mathcal{G}.\mathbf{mod},\\
{}_{R_s}\mathbf{G}.\mathbf{mod} \preceq {}_{R}\mathbf{G}.\mathbf{mod} &\preceq {}_{R_w}\mathbf{G}.\mathbf{mod},\\
{}_{R_s}\mathcal{G}.\mathbf{mod} \preceq {}_{R}\mathcal{G}.\mathbf{mod} &\preceq {}_{R_w}\mathcal{G}.\mathbf{mod}.
\end{aligned}$$

4 Conclusion

We considered the influence of hyperstructures theory on category theory and introduced some generalizations of the well-known categories (e.g. pre-semiadditive, semiadditive, pre-additive, additive, etc. categories) in which the class of all morphisms between two objects forms an algebraic structure. In this approach, first we introduced and formulated such new categories and then gave some examples of these categories. As a main example, we proceeded to introduce and study the category of general Krasner hypermodules with both single-valued and multi-valued homomorphism. For completeness, we stated some properties of morphisms in this category in [30, 31]. In the study of this category, we encounter two complexities named hyperoperation and multi-valued homomorphism. This fact makes some difficulties in investigating some categorical structures such as kernel, cokernel, equalizer, coequalizer, etc. So, this area can be a suggested field of research in future.

Acknowledgements

The second author partially has been supported by "Algebraic Hyperstructure Excellence(AHETM), Tarbiat Modares University, Tehran, Iran" and "Research Center in Algebraic Hyperstructures and Fuzzy Mathematics, University of Mazandaran, Babolsar, Iran".

The work of third author presented in this paper was supported within the project for "Development of basic and applied research developed in the long term by the departments of theoretical and applied bases FMT (Project code: DZRO K-217) supported by the Ministry of Defence the Czech Republic.

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